Chapter 3 The Time Value of Money

Chapter Overview

The *What Companies Do* opening feature shows the very applied and practical nature of the topic of this Chapter. Financial analysis focuses on (future) time and uncertainty. These concepts are the reasons the analysis is useful, but also show the inherent challenges for an analyst in trying to build a framework for evaluating what has yet to be. A time value of money analysis can help determine this.

***What Companies Do* Discussion Questions:**

1. What pieces of information may be difficult to obtain in order to create a good set of measures of the time value of money?

This is a very important 'tools' chapter, introducing valuation methods that you will use heavily in later chapters. You will have difficulty solving problems in later chapters if you do not master the techniques introduced in this chapter.

This chapter discusses:

3-1. Introduction to Time Value of Money
3-2. Future Value of a Lump Sum Received Today
3-3. Present Value of a Lump Sum Received in the Future
3-4. Additional Applications Involving Lump Sums
3-5. Future Value of Cash Flow Streams
3-6. Present Value of Cash Flow Streams
3-7. Advanced Applications of Time Value

Technology

1. **Smart Concepts** explains time value of money concepts step by step.
2. **Smart Solutions** provides solutions to Problem 3-9, and 3-15.

After studying this chapter you will be able to:

- understand how to find the future value of a lump sum invested today
- calculate the present value of a lump sum to be received in the future
- find the future value of cash flow streams, both mixed streams and annuities
- determine the present value of future cash flow streams, including mixed streams, annuities and perpetuities
- apply time-value techniques that account for compounding more frequently than annually, stated versus effective annual interest rates, and deposits needed to accumulate a future sum
- use time-value techniques to find implied interest or growth rates for lump sums, annuities and mixed streams, and an unknown number of periods for both lump sums and annuities.

Lecture Guide

Chapter 3 introduces students to time value of money tools including present value, future value, annuities, uneven cash flows, loan amortisations and the differences between effective and nominal interest rates.

- This chapter provides an opportunity to make mathematical use of the opening story. When the class is covering annuities, you can use the information in the story to calculate the

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breakeven discount rate – at what discount rate you would be better off taking the lump sum vs. the annuity.

- In this case, with the limited amount of information you will have to plug in some practice variables to see if the city made a wise decision. For example, Y = 75; PMT = (3* 39000); FV = 0; PV = ?

3-1 Introduction to the Time Value of Money

This chapter continues the theme of marginal benefits and costs. A project has an initial cost and then a stream of benefits – cash flow – in the future. The manager must decide if the future benefits will justify the initial cost of the project.

3-2 Future Value of a Lump Sum Received Today

In order to calculate future value, you need to know how much interest the money will earn, how long it will be earning interest and whether the money will be compounded annually or at an interval other than annually.

- In 1626, Peter Minuit bought Manhattan Island, now in New York, from the native American tribe living on the island for $24. You might think that the Americans got a bad deal, but if they had invested the money at 10% a year, the value of the $24 today would be $24 x $(1.1)^{375}$ = $80 trillion. This is enough to buy the world! This is an example of how powerful the time value of money is.

One of the main principles of finance is that a dollar today is worth more than a dollar tomorrow. This is because a dollar invested will earn interest and be worth more in the future. There is a tradeoff between dollars today and dollars received in the future and the value of that trade off is determined by the interest rate.

- *The future value factor, or compounding factor, is $(1 + r)^n$.*

3-2a The Concept of Future Value

Note that in the first year, you are simply multiplying $100 x 5% interest = $5. The value of $100 after one year is the principal amount of $100 plus one year's interest of $5. Note that you cannot make so simple a calculation when you move to year two.

Compound Interest

- The value of $100 in two years is not $100 + $5 + $5 = $110. Instead, the value is $110.25. This is because you have earned interest on the first year's interest of $5. So, the value in the second year is $100 in principal plus $5 in simple interest plus $5 x 5% = $0.25, the compounded interest. This is just a small difference (25 cents) in year 2, but it grows larger each year. In year 3, the simple interest is $5 x 3 = $15. Total interest is $115.76 - $100 = $15.76. Compounded interest is $15.76 - $15.00 = .76. The interest as a result of compounding has not just doubled the first year's 25 cents = it has more than tripled to $.76. The relationship between interest rate and future value is not a simple linear relationship – it is an exponential relationship. What about the interest rate?

3-2b The Equation for Future Value

Equation 3.1: $FV = PV \times (1 + r)^n$

- This formula breaks down the simple interest received in the previous example. And makes some key points:
- *Note that future value interest factors are always greater than 1* – a dollar today earns interest, and is worth more in the future because of the compounded interest it receives.
- *The greater the interest rate, the greater the future value factor will be*, *and thus the larger the future value*. This is because each dollar earns more interest and more interest on the interest received.

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- *The greater the time period, the larger the future value factor and the larger the future value*. This is because the longer you can allow money to accumulate interest, the greater the future value will be.

Albert Einstein once said the most important mathematical invention of the time was compound interest. This slide illustrates the exponential relationship between future value and time and interest rate.

- Student Involvement: Ask students which line they would like to represent their retirement savings plan.
  - Most students will choose the 20% interest rate line in which $1 has grown to almost $40 in 20 years.
- Ask them what kind of investments they would need to make to earn 20% on their money. What is a more realistic return for a retirement portfolio? This is an opportunity to begin to talk about the risk and return relationship, one of the most common themes in a finance course.
  - If an investor takes more risk, they have a higher expected return and vice versa. So, a student who wants his or her retirement fund to grow faster generally must either invest for a longer time period, or must invest in riskier, higher return assets.

Note that the 0% interest rate line is a horizontal line in Figure 3.2. This represents returns if there were no interest. Think how much could be eliminated from a finance class if interest did not exist!

Figure 3.1 Future Value of $100 Invested for Five Years at 6% Annual Interest

3-2c A Graphic View of Future Value

Sometimes it is useful for the students to see the power of compounding (and the inverse operation of discounting) through a picture. Figure 3.2 is a good example.

Figure 3.2 The Power of Compound Interest: Future Value of $1 Invested Today at Different Annual Interest Rates

3-3 Present Value of a Lump Sum Received in the Future

In finance, we use the term discounting to describe the process of calculating present values. This process answers the question what will I be willing to pay today to receive x dollars in the future.

3-3a The Concept of Present Value

- Note that by algebraically rearranging the future value equation, you have the present value equation. These equations are inverses of each other. Another name for finding the future value is compounding. Finding the present value is called discounting.
- The discount rate, r, is sometimes called the opportunity cost of money. It is the return forgone because you chose a particular investment or project.

3-3b The Equation for Present Value

- *The present value or discounting factor is* $1/(1+r)^n$.
- Note that present value factors are always less than 1. Present value is always smaller than future value. One is what you would receive today. Tomorrow a dollar is worth a little less, the day after that a little less, etc.
- The present value/future value equations find the sum of money today to which you would be indifferent at a future time at a specified interest rate.

Figure 3.3 Present Value of $1700 to be Received in Eight Years at an 8% Discount Rate

Figure 3.4 The Power of Discounting: Present Value of $1 Discounted at Different Interest Rates

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Example: Present Value of $200, 7% interest and 2 years
The value of $214 received one year from now is $200 today, or 214/1.07 = $200. The value of $228.98 received 2 years from now is $228.98/(1.07)^2 = \$200$. Another way of looking at present is noting that you need to start with less today if you have more time to earn interest. For example, suppose you want to have $200 one year from now, and you can earn 7% on your money. If you invest $200/1.07 = $186.92, you will have $200 (186.92 x 1.07) in one year. Suppose you want to have $200 two years from today and the interest rate is 7%. Now, you only need $200/(1.07)^2 = \$174.69$ today. You need less money to start with to achieve the same goal of having $200 because you now have two years to earn interest. The further out in the future that you are to receive the money, the less it is worth today.

The Power of High Discount Rates
Note again, as with future value, the relationship between present value and time and interest rate is exponential, not linear.

- The greater the interest rate, the smaller the present value factor will be and thus the smaller the present value. If the interest rate is higher, then money earns more interest over time, and you need a smaller amount of money to start with to earn a certain amount in the future.
- The greater the time period, the smaller the present value factor and the smaller the present value. This says that the more into the future that you receive money, the less it is worth today. The money has a longer period of time to earn interest.

3-3c A Graphic View of Future Value
Sometimes it is useful for the students to see the power of compounding (and the inverse operation of discounting) through a picture. Figure 3.4 is a good example.

3-4 Additional Applications Involving Lump Sums
You can also use the present value and future value formulas to find other variables such as interest rate and investment horizon. This section provides examples of finding these variables algebraically, with a calculator or spreadsheet. It also gives an excellent example of how Google appreciated after its IPO in August 2004. This shows a very practical example of the use of time value of money tools on a popular company.

Figure 3.5 Using a Calculator to Find Investment Time Period

Figure 3.6 Using a Calculator to Find Rate of Return on Investment

3-5 Future Value of Cash Flow Streams
Note that this is a tools chapter. This provides the necessary mathematical formulas that can be used to solve a variety of finance problems. The students' calculator is their friend. Point out that each financial calculator is a little different, and they should spend some time becoming familiar with the workings of their particular calculator.

In teaching future value, it is generally useful to solve the same problem using the future value formula (algebraically), using a financial calculator and using Excel. It is also helpful to ask instructors teaching higher level electives which methods they require students to use. For example, if later electives require students to have financial calculators, then stress the use of these. Higher-level electives may require students to know the formulas and solve problems manually with a non-financial calculator. The real financial world makes heavy use of spreadsheets, so it is useful for students to become familiar with Excel. Also, if you are using financial calculators, point out the most common calculator errors:

1. Most calculators have a default setting of 12 payments per period, while most time value of money problems require a setting of 1 payment per period.
2. Failing to clear previous work from the calculator.
3. Having the calculator in begin mode when it should be in end mode.

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4. Put the wrong positive or negative sign on cash flows. It is important to mark outflows with a negative sign and inflows with a positive sign.
5. Note also that some financial calculators require you to input a 0 for a time value of money variable (PV, FV, PMT, I, N) not being used in the problem. If you have three variables you can solve for the fourth. Some problems are both annuities and lump sums and use all five TVM keys.

Recommend that students start out a time value of money problem by drawing a time line. It is easier to see what are the cash inflows and outflows with a time line, and even experienced time value of money problem solvers use time lines.

The instructor can illustrate solving the future value and present value of an ordinary annuity using formulas, calculator and tables.

Smart Solutions: Present the step-by-step solution to Problem 3-27, an ordinary annuity problem.

3-5a Finding the Future Value of a Mixed Stream

The future value of any stream of cash flows at the end of a specified year is merely the sum of the future values of the individual cash flows at the year's end. We often call this value the terminal value.

Frequently a problem may call for the discounting or compounding of a series of uneven cash flows. Each cash flow can be discounted or compounded separately, as shown in the slide, with the PVs or FVs added up to find the final answer. Many financial calculators have a function to allow the analyst to enter the cash flows directly and find the answer without needing to discount each individual number. Most calculators do not have a future value function.

- Note that cash flow streams can be even or uneven. If the cash flow stream is uneven, then the student must discount (or compound) each cash flow individually, using either formulas or present value tables. Or, the student must use the cash flow function on his/her financial calculator.

This problem can be solved using formulas (compounding for future value and discounting for present value) each cash flow stream individually, as illustrated in the slide. It can also be solved using the present value and future value of $1 tables and it can be solved using the NPV function on a financial calculator. On the Texas Instruments calculator model BAII+, for example, a student would input:

CF0 = −10,000
C01 = 3,000
F01 = 1
C02 = 5,000
F02 = 1
C03 = 4,000
F03 = 1
C04 = 3,000
F04 = 1
C05 = 2,000
F05 = 1
CPT NPV
I = 4
CPT
The calculator then provides the present value of $5,271.7

Most calculators do not have a NFV function. A student must calculate each future value individually, or the student can calculate present value using a calculator, and then multiplying present value by 1 plus the discount rate raised to the nth power:

$FV = PV \times (1+r)^n = 5{,}271.65 \times (1.04)^5 = \$6{,}413.77$

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Or, the future value of this present value is:

PV = –5,271.7
I = 4
N = 5
PMT = 0
Solve for FV, FV = $6,413.8

Figure 3.7 Future Value at the End of Five Years of a Mixed Cash Flow Stream Invested at 9%

3-5b Types of Annuities

This section identifies the difference in value for an annuity depending on whether payments are made at the start or the end of each period. An annuity due is paid at the start of each period, but an ordinary annuity is paid at the end of each period. Clearly an annuity due with the same payment per period over the same number of periods as an ordinary annuity, will be worth more.

3-5c Finding the Future Value of an Ordinary Annuity

- *The instructor can solve this problem using several methods.* It is worthwhile to substitute actual numbers and note that the answer is like the future value factors found in the future value annuity table in the appendix of the textbook:

$$\frac{(1+r)^n - 1}{r} = \frac{(1.055)^5 - 1}{0.055} = 5.5811$$

FV = $1,000 $\times$ 5.5811 = $5,581.10

Most future value tables only include whole numbers for interest rates. The future value of $1, at a 5% interest rate after 5 years is 5.5256. The future value of $1, at a 6% interest rate after 5 years is 5.6371. Note that 5.5811 is about halfway between the 5% factor and the 6% factor.

Using a financial calculator to solve this problem:
PMT = 1,000
I = 5.5
N = 5
PV = 0
Solve for FV, $5,581.09

It may also be useful to go through this number-by-number, so that students understand how the final answer is reached. Point out that the future value tables, formulas and financial calculators provide shortcuts so they don't have to do this every time they solve a problem.

For example: $1,000 taken 4 periods in the future (It may be helpful at this point to count the periods out loud, showing that even though it is a five year problem, since you are receiving the first $1,000 in year 1, you only have to move ahead 4 periods into the future to reach period 5.)

For the $1,000 received in year 1:
1,000 $\times$ 1.055 $\times$ 1.055 $\times$ 1.055 $\times$ 1.055 or 1,000 $\times$ $(1.055)^4$ = $1,238.82

For the $1,000 received in year 2:
1,000 $\times$ 1.055 $\times$ 1.055 $\times$ 1.055 or 1,000 $\times$ $(1.055)^3$ = $1,174.24

For the $1,000 received in year 3:
1,000 $\times$ 1.055 $\times$ 1.055 = 1,000 $\times$ $(1.055)^2$ = $1,113.02

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For the $1,000 received in year 4:
1,000 $\times$ 1.055 = $1,055.00

For the $1,000 received in year 5, this is already in year 5 dollars, so no additional calculations need to be done. Adding up 1,238.82 + 1,174.24 + 1,113.02 + 1,055 + 1,000 = $5,581.08

Figure 3.8 Future Value at the End of Five Years of an Ordinary Annuity of $1000 per Year Invested at 7%

3-5d Finding the Future Value of Annuity Due
Students should distinguish between ordinary annuities and annuities due. Using formulas, the value of an annuity due is simply the value of the ordinary annuity multiplied by one plus the interest rate. This can be done easily on a financial calculator by toggling between the begin key for annuities due and the end key for ordinary annuities.

Here, simply multiply the answer to the previous problem, $5,581.08 x 1.055 = $5,888.04 or set the financial calculator at BGN mode.

Figure 3.9 Future Value at the End of Five Years of an Annuity Due of $1000 per Year Invested at 7%

3-6 Present Value of Cash Flow Streams

3-6a Finding the Present Value of a Mixed Stream

Figure 3.10 Present Value of a Five-Year Mixed Stream Discounted at 9%

3-6b Finding the Present value of an Ordinary Annuity
(5 years, 5.5% interest) example:

Like the future value of an annuity problem, this problem can be solved in several ways. The formula for the present value of $1 received each year for n years is:

$$\frac{1}{r} \times \left[1 - \frac{1}{(1+r)^n}\right] = \frac{1}{0.055} \times \left[1 - \frac{1}{(1.055)^5}\right] = 4.2703$$

The answer, 4.2703 is like the present value annuity factors found in the tables in the back of the textbook. Multiply the annuity factor time the payment, in this case $1,000, to find the present value of this annuity:

$1,000 $\times$ 4.2703 = $4,270.30

Or, solve this problem using a financial calculator:
PMT = 1,000
I = 5.5%
N = 5
FV = 0
Solve for PV, $4270.28
Again, it is helpful to take the problem step by step, even though students will not have to do this for every problem they solve:

The $1,000 received in year 1 is discounted back one period, 1,000/1.055 = $947.87

The $1,000 received in year 2 is discounted back two periods, 1,000/$(1.055)^2$ = $898.45

The $1,000 received in year 3 is discounted back three periods, 1,000/$(1.055)^3$ = $851.61

The $1,000 received in year 4 is discounted back four periods, 1,000/$(1.055)^4$ = $807.22

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The $1,000 received in year 5 is discounted back five periods, $1,000/(1.055)^5$ = $765.13

Adding these up, 947.87 + 898.45 + 851.61 + 807.22 + 765.13 yields the answer, $4,270.28

Figure 3.11 Present Value of a Five-Year Ordinary Annuity Discounted at 8%

3-6c Finding the Present Value of an Annuity Due
As with the future value of an annuity due, having payments made at the beginning of the period adds one extra period of interest. As with all time value of money problems, starting one period earlier gives a higher value. The value of the annuity due is $4,270.28 x 1.055 = $4,505.15. You can also solve this by setting a financial calculator in BGN mode.

3-6d Finding the Present Value of a Perpetuity
Note the very easy formula to find present value when the stream of payments continues forever. It is simply the payment divided by the discount rate. Point out what kinds of investments this might represent, for example, preferred equity, a consol bond, or a share paying a constant dividend.

3-6e Finding the Present Value of a Growing Perpetuity
This is a formula more likely to be used than the non-growing perpetuity formula. In later chapters we will assume that dividend-paying shares grow their dividend by a constant amount. We may also value companies using this formula, assuming that at some point the cash flows will continue indefinitely at a constant rate. Again, the mathematics are easy when you know the cash flow, the discount rate and the growth rate. Tell students to make a note of the fact that the formula uses the cash flow in period 1 to find the present value in year 0. The present value is always one year less than the cash flow. In other words, if you were using cash flow in year 50, your answer would be present value in year 49 dollars.

3-7 Advanced Applications of Time Value

3-7a Compounding More Frequently than Annually

- It is frequently important to know how to compound or discount for a period other than one year. For example, bonds make semi-annual payments, car payments and home mortgages are made monthly, and interest on your bank account may be compounded daily or even continuously. If the compounding period is more frequent than one year, then interest is received more often, and future value is greater. When you are comparing securities with different compounding periods, it is important to have a common rate of comparison, an equivalent annual return.
- Note that paying or receiving interest more frequently than yearly can have a big impact on value. Most loans quote an AAPR or average annual percentage rate, so if you think the rate on the 10% car loan for which you are making monthly payments is really 10%, think again. You are actually paying $(1+0.10/12)^{12} - 1 = 0.105$ or 10.5%. If you own a bond that pays 10% interest, you are really receiving $(1.05)^2 = 0.1025$ or 10.25%. Compounding interest more frequently is a good thing when you are receiving the interest but very costly when you are making the payments. **Tell students that continuous compounding is used in other financial models, for example, the Black-Scholes option pricing model assumes continuous compounding.**
- The instructor may wish to show students how to solve this problem, the present value of $125,000 at 5.13% annual interest, compounded continuously. Most calculators have a key to solve problems raising e to a power; however, the placement of this key differs on various calculators. The instructor might poll students to find the most common calculators used and have students familiar with each kind of calculator illustrate this in class. Many students will have used this function in previous math courses.

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Table 3.1 The Future Value From Investing $100 at 8% Interest Compounded Semiannually Over Two Years

Table 3.2 The Future Value From Investing $100 at 8% Interest Compounded Quarterly Over Two Years

3-7b Stated versus Effective Annual Interest Rates

- Note that r in the formula is the non-compounded rate, or average annual percentage rate. This is the rate quoted in newspapers and by lenders as 'the interest rate.' The EAR, or effective rate, or compounded rate is what you are actually paying (or receiving) in annual interest. This formula is not necessary if everything is compounded annually, which is what the present value tables and the tables built into calculators and Excel assume. It is a very useful formula if you need to compare interest rates, for example, if you are given the AAPR for one loan with quarterly compounding to another with monthly compounding.

- Here you are receiving interest, and more frequent compounding is a benefit. With quarterly compounding you have earned 9 cents more than if compounding had been annual.

- Using the formula, quarterly compounding yields higher actual returns the more frequent the compounding.

3-7c Calculating Deposits Needed to Accumulate a Future Sum

- A very practical need for TVM problems is the need to determine the annual deposits necessary to accumulate a certain amount of money at some point in the future. A very real example of this is found on Yahoo Finance – Personal Calculators. They allow the user to see how much they must deposit monthly, yearly or any other time frame to see accumulating a necessary retirement figure. This can also be done for any other type of investment – down payment on a home or car or holiday property.

3-7d Loan Amortisation

- Loan amortisation refers to the process in which a borrower makes equal periodic payments over time to fully repay a loan. This is often used in banking and credit industries.

Table 3.3 Loan Amortisation Schedule, $25,000 Principal, for 8% Interest, Five-Year Repayment Period

Chapter Summary

- This chapter relates to one of the basic decisions a manager must make. Managers invest their company's money in assets. They want to know what is the value today of the cash flows to be received in the future.
  - Ask students which choice they would make – would they prefer $1,000 today or $1,500 today. This choice is easy; everyone would choose the higher sum of money. Then ask if they would prefer $4,000 in two years or $5,000 in three years. This choice is harder to make. Now they need to have an interest rate to be able to calculate the value of each sum today in order to make this determination. You need to make these calculations because an interest rate applies.
  - Ask one student if he/she would like all the money in his/her wallet, or all the money in the wallet plus $10, no strings attached. People will say they prefer more money. Ask another student the same question, and you usually receive the same answer. The students have just demonstrated the economic principle that more is preferred to less. Ask students for an example in the real world when they would not desire more of a desirable thing. Many will give answers such as chocolate, ice cream, food when you've already had a big meal, etc. Tell students that in finance we will work with money and assume that more is preferred.

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Now ask how you can choose between the students since you only have one $10 note. You may get answers related to who needs it the most, who has the most money in his wallet now, etc. Tell the students that this is the basic decision that companies have to make. Do companies pay out their earnings as dividends (consume) or retain within the company (save)? Companies don't have to worry about who needs the money more, since as long as they follow the rule to accept all positive net present value projects, they will be maximising shareholder wealth. It is then up to the consumer to decide whether to spend now and save less or save more now and spend more later. Some people will choose to spend more now and borrow – many students are in this group, spending more than their current earnings. Others will choose to save more now and spend less. Financial markets bring together borrowers and lenders with an interest rate determined by supply and demand for money.

Enrichment Exercises

1. Show students a transparency of a quote from Stewart Myers, 'The opportunity cost of capital depends on the use of funds, not on the source.' Ask students the meaning of this quote. Finance theory stresses cash flow and expected return on assets. The company's possible investments compete with other securities that shareholders can buy. Investors will only invest in the company if they believe the company will earn a higher risk-adjusted return on its investments than the investors can do on their own.
2. Ask student to prove whether time travel is possible. The existence of an interest rate shows that time travel cannot exist. Suppose a time traveller puts $1,000 into a savings account earning 10% interest. In 10 years, this will be worth $2,594. The time traveller could zip into the future, withdraw his money, zip back to the past, keep $1,594 and reinvest the $1,000, and start the whole process again. The time machine would be a money machine. If everyone had access to time travel, the world would prosper. Since there is a positive interest rate, time travel is not possible.

3. If students are having difficulty using their financial calculators, refer them to http://clem.mscd.edu/~mayest/calculators/calculator_tutorials.index.htm.

4. Another source, as an instructor or student reference, is Womack, Kent L. and Andrew Brownell, 'Financial Math on Spreadsheet and Calculator.' This article gives step-by-step instructions concerning time value of money, future value, present value, annuities, perpetuities, and pricing bonds, using calculators and spreadsheets.

5. Introduce students to the financial tools on Excel. Show them how to put in all the variables and use the Excel function tools to complete the TVM problems.

6. Have the students visit Yahoo Finance/ Personal Finance/ Calculators and see how much they need to accumulate in order to retire a millionaire.

Answers to Concept Review Questions

1. Assuming a positive interest rate (i.e. $r > 0$), then $1 in the future is always worth less than $1 today because the present value of $1 received in the future is $\$1 / (1 + r)^N$ which results in a value below $1.

2. If the interest rate ('r') equals zero, then future value (FV) and present value (PV) are equal in value. Notice, $PV \times (1 + r) = FV$, setting 'r' to zero makes $PV = FV$.

3. Compounded interest returns more than simple interest over multiple periods because you earn interest on the interest, so usually a deposit made into an account paying compounding interest will

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yield a higher future value. For the very first period, however, the return is the same with simple or compound interest. Holding an investment longer will also mean a higher future value in the account paying compound interest, because there is more time for the investment to earn interest on both the original investment AND on the accrued interest.

4. A decrease in the interest rate would lower future value, while an increase in the holding period will increase future value. Decreasing the interest rate decreases the future value factor and thus future value. Increasing the holding period increases the future value factor and thus future value. Looking at the expression for future value, $FV = PV \times (1+r)^n$, where r is interest rate and n is the number of holding periods, should make these relationships obvious.

5. Present values and future values of a lump sum are related because the future value of an amount of money is equal to its present value times one plus the interest rate raised to the nth power, with n being the number of years money is being compounded. Similarly, present value is future value divided by one plus the interest rate raised to the nth power. This means that, given present value, interest and number of years, you can find the future value. Mathematically, the relationship is: $PV = FV / (1 + r)^N$ or $FV = PV \times (1 + r)^N$.

6. An increase in the discount rate will decrease the present value factor and hence the present value: A higher interest rate means you would have to set less aside today to earn a specified amount in the future. A decrease in the time period increases the present value factor and increases the present value: If you have less time, you will have to set aside more today to have a specified amount in the future.

7. Allowing annual interest of 4% to accrue over 28.6 years results in a value over \$3,000 (\$1,000(1 + 4%)$^{28.6}$ = \$3,070.11. Consequently, it takes less than 28.6 years to triple the value of \$1,000 while earning 4% annual interest (in fact, it takes 28.02 years). The reason for needing less compounding periods is because over a two year period, 4% annual compounding is equivalent to 8.16% compounding bi-annually. If the bi-annual compounded rate equalled 8% exactly, then 28.6 years would be needed to triple the value of \$1,000. However the bi-annual compounded rate is actually slightly higher than 8%, making fewer compounding periods necessary when earning 4% annually.

8. An ordinary annuity generates payments at the end of each period. Annuities due, on the other hand, generate payments at the beginning of each period. Cash flows can be converted from an ordinary annuity into an annuity due by multiplying the final answer by one plus the interest rate.

9. An ordinary annuity can easily be converted into an annuity due by multiplying the ordinary annuity value by one plus the interest rate. The main difference is that you are receiving (or paying) one more payment with an annuity due than with an ordinary annuity, since annuities due make the payments at the beginning of the month instead of the end of the month.

10. The future value of a mixed stream of cash flows would be calculated by taking each individual cash flow, and then compounding it, using future value factors or the future value formula to extend that amount into the future. We would then add up all of the future-valued amounts for the final future value of the mixed stream of cash flows.

    You could find the present value of a mixed stream of cash flows using the cash flow menu of a financial calculator or you could individually discount each cash flow to the present using present value factors or the present value formula and then sum all of the values.

11. The present value of an annuity due is equivalent to an ordinary annuity with the same inputs multiplied by: one plus the interest rate.

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12. A perpetuity does pay an infinite amount of cash; however, distant future cash flows have a present value of zero, making the value of a perpetuity a non-infinite value.

13. The value of a growing perpetuity is the cash flow in period 1 divided by: the interest rate minus the growth rate. In this case, the growth rate would be negative.

14. The effective rate equals: $(1 + r/m)^m - 1$, where 'r' is the stated annual rate. If 'm' is less than one, i.e. compounding occurs at a less than annual basis, then the effective rate will be greater than the stated annual rate.
15. The total amount paid by the borrower (i.e. interest and principal) will be less with weekly payments when compared to monthly payments. This occurs because weekly payments repay principal faster, allowing for less interest to accrue on the loan. Because less interest accrues, the total amount paid by the borrower decreases.

Answers to Self-Test Problems

ST3-1. Starratt Alexander is currently considering investing specified amounts in each of four investment opportunities described below. For each opportunity, determine the amount of money Starratt will have at the end of the given investment horizon.

Investment A: Invest a lump sum of $2,750 today in an account that pays 6% annual interest and leave the funds on deposit for exactly 15 years.

Investment B: Invest the following amounts at the beginning of each of the next five years in a venture that will earn 9% annually and measure the accumulated value at the end of exactly 5 years:

Beginning of Year	Amount
1	$900
2	1,000
3	1,200
4	1,500
5	1,800

Investment C: Invest $1,200 at the end of each year for the next 10 years in an account that pays 10% annual interest, and determine the account balance at the end of year 10.

Investment D: Make the same investment as in investment C but place the $1,200 in the account at the *beginning* of each year.

A: **Investment A**: This problem asks you to calculate the future value of a lump sum. It is a straightforward application of Equation 3.1 as follows: FV = $2,750 X $(1 + 0.06)^{15}$ = $6,590.53. In Excel, you could obtain the same value by entering, =fv(0.06,15,0,-2750,0).

Investment B: In this problem you are trying to find the future value of a mixed stream, which amounts to finding the future value of each component in the stream and adding the future values together. Algebraically, we are applying Equation 3.3 using the cash flows given in the problem and the 9% interest rate:
FV = $\$900(1.09)^5 + \$1,000(1.09)^4 + \$1,200(1.09)^3 + \$1,500(1.09)^2 + \$1,800(1.09)^1$
FV = $1,384.76 + $1,411.58 + $$1,554.03 + $$1,782.15 + $1,962.00 = $8,094.53

Investment C: This is an ordinary 10-year annuity with $1,200 annual payments. Given the 10% interest rate, we can apply Equation 3.4 to find the future value:

$$FV = \$1{,}200 \times \left\{\frac{(1 + 0.10)^{10} - 1}{0.10}\right\} = \$19{,}124.91$$

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Alternatively, we could use the Excel function for the future value of an annuity and simply enter =fv(0.10,10,-1200,0,0).

Investment D: Investment D is identical to Investment C, except that the former is an annuity due while the latter is an ordinary annuity. In other words, the cash flows associated with Investment D are the same as Investment C, except that they arrive one year earlier and can therefore earn one extra year of interest. For that reason, we know that the future value of Investment D should simply be 10% greater than the future value of Investment C. We can verify this using Equation 3.5:

$$FV = \$1{,}200 \times \left\{\frac{(1+0.10)^{10}-1}{0.10}\right\} \times (1+0.10) = \$21{,}037.40$$

We could also simply change the 'type' argument in Excel's future value function from 0 to 1 to obtain the same answer. Just enter =fv(0.10,10,-1200,0,1).

ST3-2. Gregg Snead has been offered four investment opportunities, all equally priced at $45,000. Because the opportunities differ in risk, Gregg's required returns (i.e., applicable discount rates) are not the same for each opportunity. The cash flows and required returns for each opportunity are summarised below.

Opportunity	Cash Flows		Required Return
A	$7,500 at of 5 years.		12%
B	Year	Amount	15%
	1	$10,000	
	2	12,000	
	3	18,000	
	4	10,000	
	5	13,000	
	6	9,000	
C	$5,000 at the *end of each year* for the next 30 years.		10%
D	$7,000 at the *beginning of each year* for the next 20 years.		18%

a. Find the present value of each of the four investment opportunities.
b. Which, if any, opportunities are acceptable?
c. Which opportunity should Gregg take?

A: a. *Investment opportunity A* is an ordinary 5-year annuity paying $7,500 per year. Applying Gregg's 12% discount rate and Equation 3.7 we have:

$$PV = \frac{\$7{,}500}{0.12} \times \left[1 - \frac{1}{(1+0.12)^5}\right] = \$27{,}035.82$$

We could also use Excel's present value function, entering, =pv(0.12,5,-7500,0,0).

Investment opportunity B is a mixed stream. To find the present value we use Equation 3.6 and plug in the cash flows and the 15% required return:

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$$PV = \frac{\$10{,}000}{(1+0.15)^1} + \frac{\$12{,}000}{(1+0.15)^2} + \frac{\$18{,}000}{(1+0.15)^3} + \frac{\$10{,}000}{(1+0.15)^4} + \frac{\$13{,}000}{(1+0.15)^5} + \frac{\$9{,}000}{(1+0.15)^6}$$

$$PV = \$8{,}695.65 + \$9073.72 + \$11{,}835.29 + \$5{,}717.53 + \$\$6{,}463.30 + \$3{,}890.95 = \$45{,}676.45$$

We could also use Excel's net present value function and simply enter the required return followed by each of the cash flows, =npv(0.15,10000,12000,18000,10000,13000,9000). Note that because we entered the cash flows here with a positive sign, Excel would produce a negative value for the answer (–$45,676.45).

Investment opportunity C is a 30-year annuity paying $5,000 annually. We can use Equation 3.7 to find the present value:

$$PV = \frac{\$5{,}000}{0.10} \times \left[1 - \frac{1}{(1+0.10)^{30}}\right] = \$47{,}134.57$$

You can find the same answer by entering into Excel, =pv(0.10,30,-5000,0,0).

Investment opportunity D is a 7-year annuity due paying $7,000 annually. Plug in the 18% required return and the other inputs into Equation 3.8 to find the present value:

$$PV = \frac{\$7{,}000}{0.18} \times \left[1 - \frac{1}{(1+0.18)^{20}}\right] \times (1+0.18) = \$44{,}213.69$$

In Excel, we could use the present value function to obtain the same value by entering =pv(0.18,20,-7000,0,1).

b. An investment is acceptable if the present value of the cash flows that Gregg will receive is larger than the cost of buying the investment. Only Investments B and C satisfy that condition, so they are acceptable and the other opportunities are not.

c. Gregg should take Investment C because its present value exceeds that of Investment B, yet both investments cost the same.

ST3-3. Assume you wish to establish a university scholarship of $2,000 per year for a deserving student at the high school you attended. You would like to make a lump-sum gift to the high school to fund the scholarship into perpetuity. The school's treasurer assures you that they will earn 7.5% annually forever.

a. How much must you give the high school today to fund the proposed scholarship program assuming the first scholarship is awarded next year?
b. If you wanted to allow the amount of the scholarship to increase annually after the first award (end of year 1) by 3% per year, how much must you give the school today to fund the scholarship program?
c. Compare, contrast, and discuss the difference in your response to parts a and b.

A: a. The present value of the proposed perpetuity is $2,000/.075 = $26,667

b. The present value of the growing perpetuity is $2,060/(0.075–0.03) = $2,060/.045 = $45,778

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a. The amount that I need to give the high school if I want the scholarship to grow at 3% per year indefinitely, assuming they will be able to earn the proposed interest rate, is almost double the amount needed if the scholarship does not grow. This effect is due to the fact that we discount the annual cash flow by a smaller number in order to account for the annual growth in the scholarship.

ST3-4. Assume that you deposit $10,000 today into an account paying 6% annual interest and leave it on deposit for exactly 8 years.

a. How much will be in the account at the end of 8 years if interest is compounded:
1. annually?
2. semiannually?
3. monthly?
4. continuously?

b. Calculate the effective annual rate (EAR) for a (1) through a (4) above.

c. Based on your findings in parts a and b, what is the general relationship between the frequency of compounding and EAR?

A: a. In this question we will repeatedly apply Equation 3.12, which we repeat here:

$FV = PV \times \left(1+\frac{r}{m}\right)^{m \times n}$ In all parts of this problem, PV = $10,000, n = 8, and r = 0.06. Only m varies from one part of the problem to another. For annual compounding, m = 1, and FV = $\$10{,}000(1.06)^{8}$ = $15,938.48. For semiannual compounding, m = 2, and FV = $\$10{,}000(1.03)^{16}$ = $16,047.06. For monthly compounding, m = 12, and FV = $\$10{,}000(1.005)^{96}$ = $16.141.43. Finally, for continuous compounding we must use Equation 3.13, which gives
FV = $\$10{,}000e^{(0.06)(8)}$ = $16,160.74.

b. In this question we will repeatedly apply Equation 3.14 which appears below:

$$EAR = \left(1+\frac{r}{m}\right)^{m} - 1$$

In each scenario, r = 0.06, but the value of m changes with different compounding intervals. With annual compounding, m = 1 and the effective rate and the stated rate are both 6%. With semiannual compounding, m = 2, so the effective rate is $(1.03)^2 - 1$ = 0.0609 or 6.09%. With monthly compounding, m = 12, and the effective rate becomes $(1.005)^{12} - 1$ = 0.0617 or 6.17%. Finally, with continuous compounding we need to use Equation 3.14a, and the effective rate is $e^{0.06} - 1$ = 0.0618 or 6.18%.

c. In general, the more frequently interest compounds, the higher is the future value and the higher is the effective annual rate of return on the investments.

ST3-5. Imagine that you are a professional personal financial planner. One of your clients has asked you the following two questions. Use the time-value-of-money techniques to develop appropriate responses to each question.

a. I need to save $37,000 over the next 15 years to fund my 3-year-old daughter's university education. If I made equal annual end-of-year deposits into an account that earns 7% annual interest, how large must this deposit be?

b. I borrowed $75,000 and am required to repay it in 6 equal (annual) end-of-year instalments of $16,718.98 and want to know what interest am I paying?

A: a. The client wants to create a 15-year annuity earning 7% which will ultimately have a future value of $37,000. The unknown quantity here is the payment. We can solve for the payment using Equation 3.15:

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$$PMT = \frac{\$37{,}000}{\left[\frac{(1+0.07)^{15}-1}{0.07}\right]} = \$1{,}472.40$$

We could also use the payment function in Excel to obtain the answer. Into any cell in a spreadsheet just enter, =pmt(0.07,15,0,-37000,0).

b. Here again the client has an annuity. This one lasts 6 years, has an annual payment of $16,718.98, and has a present value of $75,000. The unknown variable is the interest rate. Problems that ask you to solve for the interest rate can be tricky to solve algebraically, so you will typically use a financial calculator or Excel to find the solution. In Excel, just type, =rate(6,--16718.98,75000,0,0) and you will obtain the answer, 9%.

Answers to End-of-Chapter Questions

Q3-1. What is the importance to an individual of understanding time value of money concepts? For a company manager? Under what circumstance would the time value of money be irrelevant?

A3-1. An individual would want to know time value of money techniques in order to compare investments. Which is better – shares, bonds, preferred equity, real estate, etc.? A company manager uses time-value-of-money techniques to make accept/reject decisions for the company's projects. An example of where time-value-of-money might not be used is if a company is required to take on a specified project (such as one that addressed an environmental or safety issue). Then time-value- of-money would not matter, since the company would have to make the investment to comply with government rules.

Q3-2. Actions that maximise profit may not maximise shareholder wealth. What role can the time value of money play in explaining the discrepancy between maximising profits and maximising value?

A3-2. Maximising profits might not be the same as maximising shareholder wealth. Profits are an accounting number that can be easily manipulated. Usually profits and cash flows are highly correlated, but this does not necessarily have to be the case.

Q3-3. You are considering two investment plans. Plan A requires you to save $100 per month for 10 years. Plan B requires you to save $200 per month for 5 years. Assuming that both plans earn the same rate of return, which plan accumulates more money?

A3-3. At any positive interest rate, Plan A results in a higher future value. Compound interest is the reason why. Both plans involving contributing the same amount of money, but the money in Plan A is invested longer and will earn more interest.

Q3-4. In the US, most government lotteries pay out jackpots in the form of a twenty- or thirty-year annuity, but they also give winners the option to collect their winnings as a much smaller lump sum. Explain how you would use time value of money analysis to choose between the annuity and the lump sum if you won the lottery.

A3-4. A lottery winner should consider what return they could earn if they accepted the lump sum and invested it. Then the winner should use that rate of return as the discount rate to calculate the present value of the annuity. If the present value of the annuity is greater than the lump sum, then take the annuity. Otherwise, accept the lump sum.

Q3-5. What happens to the present value of a cash-flow stream when the discount rate increases? Place this in the context of an investment. If the required return on an investment goes up but

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the expected cash flows do not change, would you be willing to pay the same price for the investment or would you pay more or less for this investment than before interest rates changed?

A3-5. The present value of a cash flow stream decreases when the interest rate increases. If interest rates increase, the future cash flows are worth less and you would be willing to pay less for the investment.

Q3-6. Look at the formula for the present value of an annuity. What happens to the present value as the number of periods increases? What distinguishes an annuity from a perpetuity? Why is there no formula for the future value of a perpetuity?

A3-6. As the number of periods increases, the present value increases. You are receiving more payments and adding to present value. An annuity last for a finite number of years, while a perpetuity lasts forever. There is no future value formula for a perpetuity because infinity is not a specified time into the future.

Q3-7. Suppose you borrow a large sum of money to buy a house, and you will pay back the loan over thirty years making fixed monthly payments. After fifteen years have passed, will you have paid off half the loan principal, more than half, or less than half? Why?

A3-7. In the early years, most of the monthly payment goes to pay interest, not principal. The percentage of each payment going to principal increases over time, but after 15 years you would have paid off less than one-half of the original loan principal.

Q3-8. Under what circumstances is the effective annual rate different than the stated annual rate, and when are they the same?

A3-8. The effective rate is greater than the stated rate as long as interest compounds more frequently than once per year. If interest compounds once per year, the two are identical.

Solutions to End-of-Chapter Problems

Future Value of a Lump Sum Received Today

P3-1. You have \$1,500 to invest today at 7% interest compounded annually.

a. How much will you have accumulated in the account at the end of the following number of years?
   1. three years
   2. six years
   3. nine years

b. Use your findings in part (a) to calculate the amount of interest earned in
   1. years 1 to 3
   2. years 4 to 6
   3. years 7 to 9

c. Compare and contrast your findings in part (b). Explain why the amount of interest earned increases in each succeeding 3-year period.

A3-1. Future Value: $FV_n = PV \times (1 + r)^n$ or $FV_n = PV \times (FVF_{r\%,n})$

a. 1. $FV_3 = PV \times (1.07)^3$

$FV_3 = \$1,500 \times (1.22504)$

$FV_3 = \$1,837.57$

b. 1. Interest earned = $FV_3 - PV$

Interest earned = \$1,837.57 − 1,500.00 = \$ 337.57

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2. $FV_6 = PV \times (1.07)^6$
$FV_6 = \$1,500 \times (1.50073)$
$FV_6 = \$2,251.10$

2. Interest earned = $FV_6 - FV_3$
Interest earned = $2,251.10
−1,837.57
$ 413.53

3. $FV_9 = PV \times (1.07)^9$
$FV_9 = \$1,500 \times (1.83846)$
$FV_9 = \$2,757.69$

3. Interest earned = $FV_9 - FV_6$
Interest earned = $2,757.69
−2,251.10
$ 506.59

c. The fact that the longer the investment period the larger the total amount of interest collected is not unexpected and is due to the greater length of time that the principal sum of $1,500 is invested. The most significant point is that the incremental interest earned per 3 year period increases with each subsequent 3-year period. The total interest for the first 3 years is $337.57; however, for the second 3 years (from year 3 to 6) the additional interest earned is $413.93. For the third 3-year period the incremental interest is $506.19. This increasing change in interest earned is due to compounding, the earning of interest on previous interest earned. The greater the previous interest earned the greater the impact of compounding.

P3-2. Dixon Shuttleworth has a large sum of money that he wants to invest to finance his retirement. He has been presented with three options. The first investment offers a 5% return for the first 5 years, a 10% return for the next 5 years, and a 20% return thereafter. The second investment offers 10% for the first 10 years and 15% thereafter. The third investment offers a constant 12% rate of return. Determine which of these investments is the best for Dixon if he plans to retire in the following number of years:

a. 15 years
b. 20 years
c. 30 years

A3-2. a. Investment # 1: Future Value Factor = $(1.05)^5 \times (1.10)^5\ x\ (1.20)^5 = 5.115$

Investment # 2: Future Value Factor = $(1.10)^{10} \times (1.15)^5 = 5.217$

Investment # 3: Future Value Factor = $(1.12)^{15} = 5.474$

If Shuttleworth retires in 15 years, investment #3 has the highest future value.

b. Investment # 1: Future Value Factor = $(1.05)^5 \times (1.10)^5 \times (1.20)^{10} = 12.727$

Investment # 2: Future Value Factor = $(1.10)^{10} \times (1.15)^{10} = 10.493$

Investment # 3: Future Value Factor = $(1.12)^{20} = 9.646$

If Shuttleworth retires in 20 years, investment # 1 has the highest future value.

c. Investment # 1: Future Value Factor = $(1.05)^5 \times (1.10)^5 \times (1.20)^{20} = 78.802$

Investment # 2: Future Value Factor = $(1.10)^{10} \times (1.15)^{20} = 42.451$

Investment # 3: Future Value Factor = $(1.12)^{30} = 29.959$

If Shuttleworth retires in 30 years, investment # 1 has the highest future value.

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Present Value of a Lump Sum Received in the Future

P3-3. A state savings bond from New South Wales can be converted to $100 at maturity six years from purchase. If the state bonds pay 8% annual interest (compounded annually), at what price must the state sell its bonds? Assume no cash payments on savings bonds before redemption.

A3-3. Present Value: $PV = FV_n \times \left[\frac{1}{(1+r)^n}\right]$ or $FV_n \times (PVIF_{r\%,\,n})$

$PV = \$100 \times (1.08)^{-6}$

$PV = \$100 \times (.6302)$

$PV = \$63.02$

P3-4. You have a trust fund that will pay you $1 million exactly 10 years from today. You want cash now, so you are considering an opportunity to sell the right to the trust fund to an investor.

a. What is the least you will sell your claim for if you could earn the following rates of return on similar-risk investments during the 10-year period?
   1. 6 %
   2. 9 %
   3. 12 %

b. Rework part (a) under the assumption that the $1 million payment will be received in 15 rather than 10 years.

c. Based on your findings in parts (a) and (b), discuss the effect of both the size of the rate of return and the time until receipt of payment on the present value of a future sum.

A3-4. a. (1) $PV = \$1{,}000{,}000 \times (1.06)^{-10}$
$PV = \$1{,}000{,}000 \times (0.558395)$
$PV = \$558{,}395$

(2) $PV = \$1{,}000{,}000 \times (1.09)^{-10}$
$PV = \$1{,}000{,}000 \times (0.422411)$
$PV = \$422{,}411$

(3) $PV = \$1{,}000{,}000 \times (1.12)^{-10}$
$PV = \$1{,}000{,}000 \times (0.321973)$
$PV = \$321{,}973$

b. (1) $PV = \$1{,}000{,}000 \times (1.06)^{-15}$
$PV = \$1{,}000{,}000 \times (0.417265)$
$PV = \$417{,}265$

(2) $PV = \$1{,}000{,}000 \times (1.09)^{-15}$
$PV = \$1{,}000{,}000 \times (0.274538)$
$PV = \$274{,}538$

(3) $PV = \$1{,}000{,}000 \times (1.12)^{-15}$
$PV = \$1{,}000{,}000 \times (0.182696)$
$PV = \$182{,}696$

c. As the rate of return increases, the present value becomes smaller. This decrease arises from the higher opportunity cost associated with the higher rate. Also, the longer the time until the lottery payment is collected, the less the present value due to the greater time over which the opportunity cost applies. In other words, the larger the rate of return and the longer the time until the money is received, the smaller will be the present value of a future payment.

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Additional Applications Involving Lump Sums

P3-5. You have saved $10,000 toward a down payment on a home. The money is invested in an account earning 7% interest. You will be ready to purchase the new home once your savings account grows to $25,000.
a. Approximately how many years will it take for the account to reach $25,000?
b. If the interest rate doubles to 14%, how many years will pass before you reach your $25,000 target?

A3-5. a. You are trying to solve this equation for 'n': $25,000 = $10,000$(1.07)^n$. You can use natural logarithms as follows:
$(25{,}000/10{,}000) = (1.07)^n$
$\text{Ln}(2.5) = n\text{Ln}(1.07)$
n=13.5 years.

You could find the same answer by entering the following formula in Excel:
=nper(0.07,0,10000,-25000,0) = 13.5 years.

b. Use the methods in part (a) to solve for the case when the interest rate is 14%. The answer is 7 years, a little more than half the time required in part (a). Intuitively, you might think that at twice the rate it will take half the time to reach the goal, but that is not quite right. With compound interest, the account balance grows very rapidly the longer the money is invested, so there is an advantage to leaving money invested longer. As a result, doubling the interest rate doesn't cut in half the time needed to reach the goal.

P3-6. You purchased a home for $250,000 eight years ago, and now the home is worth $300,000. What annual rate of return did you earn on your home?

A3-6. $[\$300{,}000 \div \$250{,}000]^{1/8} - 1 = 2.31\%$

P3-7. Find the rates of return required to do the following:
a. Double an investment in 4 years
b. Double an investment in 10 years
c. Triple an investment in 4 years
d. Triple an investment in 10 years

A3-7. $FV = PV \times (1 + r)^4$

a. $2.0 = (1 + r)^4$
$(2)^{1/4} = (1 + r)$
$1.189207 = 1 + r$
$r \approx 18.92\%$

b. $2.0 = (1 + r)^{10}$
$(2)^{1/10} = (1 + r)$
$1.071773 = 1 + r$
$r \approx 7.18\%$

c. $3.0 = (1 + r)^4$
$(3)^{1/4} = (1 + r)$
$1.31607 = 1 + r$
$r \approx 31.61\%$

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d. $3.0 = (1+ r)^{10}$
$(3)^{1/10} = (1 + r)$
$1.116123 = 1 + r$
$r = 11.61\%$

P3-8. Determine the length of time required to double the value of an investment, given the following rates of return.

a. 4 %
b. 10 %
c. 30 %
d. 100 %

A3-8. a. $FV = PV \times (1 + r)^n$
$2.0 = (1.04)^n$
$\log 2 = n\log 1.04$
$0.301 = n \times 0.017$
$n = 17.7$ years

b. $2.0 = (1.1)^n$
$\log 2 = n\log 1.1$
$0.301 = n \times 0.0414$
$n = 7.27$ years

c. $2.0 = (1.3)^n$
$\log 2 = n\log 1.3$
$0.301 = n \times 0.1139$
$n = 2.64$ years

d. $2.0 = (2)^n$
$n = 1$ year

P3-9. The viatical industry offers a rather grim example of present value concepts. A company in this business, called a viator, purchases the rights to the benefits from a life insurance contract from a terminally ill client. The viator may then sell claims on the insurance payout to other investors. The industry began in the early 1990s as a way to help AIDS patients capture some of the proceeds from their life insurance policies for living expenses.

Suppose a patient has a life expectancy of 18 months and a life insurance policy with a death benefit of $100,000. A viator pays $80,000 for the right to the benefit, and then sells that claim to another investor for $80,500.

a. From the point of view of the patient, this contract is like taking out a loan. What is the compound annual interest rate on the loan if the patient lives exactly 18 months? What if the patient lives 36 months?
b. From the point of view of the investor, this transaction is like lending money. What is the compound annual interest rate earned on the loan if the patient lives 18 months? What if the patient lives just 12 months?

A3-9. a. $80{,}000 = 100{,}000/(1+r)^{1.5}$
$r = 16\%$, if the patient lives 18 months

$80{,}000 = 100{,}000/(1+r)^3$
$r = 7.7\%$, if the patient lives 36 months

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b. $80{,}500 = 100{,}000/(1+r)^{1.5}$
r = 15.75%, if the patient lives 18 months

$80{,}500 = 100{,}000/(1+r)$
r = 24.3%, if the patient lives 12 months

Future Value of Cash Flow Streams

P3-10. Liliana Alvarez's employer offers its workers a two-month paid sabbatical every seven years. Liliana, who just started working for the company, plans to spend her sabbatical touring Europe at an estimated cost of $25,000. To finance her trip, Liliana plans to make six annual end-of-year deposits of $2,500 each, starting this year, into an investment account earning 8% interest.

a. Will Liliana's account balance at the end of seven years be enough to pay for her trip?

b. Suppose Liliana increases her annual contribution to $3,150. How large will her account balance be at the end of seven years?

A3-10. a. FV = 2,500 × FVAF (7, 8%) = $2,500 × 8.9228 = $22,307. Therefore, Liliana's balance will not be enough for her to cover the trip.

b. FV = 3,150 × 8.9228 = $28,107. In this case the account balance will be enough for Liliana to make the trip.

P3-11. Robert Williams is considering an offer to sell his medical practice, allowing him to retire five years early. He has been offered $500,000 for his practice and can invest this amount in an account earning 10 % per year. If the practice is expected to generate the following cash flows, should Robert accept this offer and retire now?

End of Year	Cash Flow
1	$150,000
2	150,000
3	125,000
4	125,000
5	100,000

A3-11. FV on original retirement date if early retirement is chosen:

$\$500{,}000 \times (1.10)^5 = \$805{,}255$

FV on retirement date if early retirement is not chosen:

$\$150{,}000 \times (1.10)^4 =$	$219,615
$\$150{,}000 \times (1.10)^3 =$	199,650
$\$125{,}000 \times (1.10)^2 =$	151,250
$\$125{,}000 \times (1.10)^1 =$	137,500
$\$100{,}000 \times (1.10)^0 =$	100,000
	$808,015

Robert Williams should not retire early because the future value of his cash flows at the end of five years would be about $3,000 less than if he continued working..

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P3-12. Gina Coulson has just contracted to sell a small parcel of land that she inherited a few years ago. The buyer is willing to pay $24,000 now. Alternatively, the buyer will make the series of payments shown in the following table, with each payment made at the *beginning* of the year. Because Gina doesn't really need the money today, she plans to let it accumulate in an account that earns 7% annual interest.

Mixed Stream

Beginning of Year (*t*)	**Cash Flow (CF_t)**
1	$ 2,000
2	4,000
3	6,000
4	8,000
5	10,000

a. What is the future value of the lump sum at the end of year 5?
b. What is the future value of the mixed stream at the end of year 5?
c. Based on your findings in parts (a) and (b), which alternative should Gina take?
d. If Gina could earn 10% rather than 7% on the funds, would your recommendation in part (c) change? Explain.

A3-12. a. $FV_5 = PV \times (1.07)^5$
$FV_5 = \$24,000 \times (1.403)$
$FV_5 = \$33,661$

b.

Beginning of Year	**Number of Years (*t*)**	**FV = $CF_t \times (1 + .07)^t$**	**Future Value**
1	5	$ 2,000 × 1.403 =	$ 2,805.10
2	4	$ 4,000 × 1.311 =	5,243.18
3	3	$ 6,000 × 1.225 =	7,350.26
4	2	$ 8,000 × 1.1449 =	9,159.20
5	1	$10,000 × 1.070 =	10,700.00
		Total =	$35,257.74

c. Gina should select the stream of payments rather than the upfront $24,000.

d. *Lump sum*
$FV_5 = PV \times (1.10)^5$
$FV_5 = \$24,000 \, x \, (1.611)$
$FV_5 = \$38,652.24$

Mixed stream

Beginning of Year	**Number of Years (*t*)**	**FV = $CF_t \times (1 + .10)^t$**	**Future Value**
1	5	$ 2,000 × 1.611 =	$ 3,221.02
2	4	$ 4,000 × 1.464 =	5,856.40
3	3	$ 6,000 × 1.331 =	7,986.00
4	2	$ 8,000 × 1.210 =	9,680.00
5	1	$10,000 × 1.100 =	11,000.00
		Total =	$37,743.42

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Note that, although the future sums of each alternative are larger at 10% than at 7%, the 10% upfront payments result in greater future value at the end of year 5 than does the mixed stream. Therefore, the upfront lump-sum payment would be preferred. This conclusion differs from that in part c primarily due to the different patterns of cash flow associated with the lump sum and mixed stream payment alternatives.

P3-13. For the following questions, assume an annual annuity of $1,000 and a required return of 12%.

a. What is the future value of a ten-year *ordinary annuity*?
b. If you earned an additional year's worth of interest on this annuity, what would be the future value?
c. What is the future value of a 10-year *annuity due*?
d. What is the relationship between your answers in parts (b) and (c)? Explain.

A3-13. a. $FVA_{10} = \$1{,}000 \times \left[\dfrac{(1.12)^{10} - 1}{0.12}\right] = \$17{,}549$

b. $\$17{,}549 \times (1.12) = \$19{,}655$

c. FVA_{10} (annuity due) $= \$1{,}000 \times \left[\dfrac{(1.12)^{10} - 1}{0.12}\right] \times (1.12) = \$19{,}655$

d. The answers to parts b and c are identical, implying that the future value of annuity due is simply the future value of an ordinary annuity plus an additional interest payment.

P3-14. Kim Edwards and Hiroshi Suzuki are both newly minted 30-year-old MBAs. Kim plans to invest $1,000 per month into her defined contribution superannuation plan beginning next month. Hiroshi intends to invest $2,000 per month into his super plan, but he does not plan to begin investing until 10 years after Kim begins investing. Both Kim and Hiroshi will retire at age 67, and their super plans average a 12% annual return. Who will have more superannuation funds available at retirement?

A3-14. Kim's future retirement account at age 67 ($r = .12/12 = .01$; $n = 37_{yrs} \times 12_{mos/yr} = 444_{mos}$):

$$FV_{37} = \$1{,}000 \times \left[\frac{(1.01)^{444} - 1}{.01}\right] = \$8{,}192{,}586$$

Hiroshi's future retirement account at age 67 ($r = .12/12 = .01$; $n = 27_{yrs} \times 12_{mos/yr} = 324_{mos}$):

$$FV_{37} = \$2{,}000 \times \left[\frac{(1.01)^{324} - 1}{.01}\right] = \$4{,}825{,}220$$

Clearly, Kim will have far more money at retirement ($8,192,586) than will Hiroshi ($4,825,220).

P3-15. To supplement your planned retirement, you estimate that you need to accumulate $220,000 in 42 years. You plan to make equal annual end-of-year deposits into an account paying 8 % annual interest.

a. How large must the annual deposits be to create the $220,000 fund in 42 years?
b. If you can afford to deposit only $600 per year into the account, how much will you have accumulated by the end of the forty-second year?

A3-15. a. $PMT = FVA_{42} \div \left[\dfrac{(1+.08)^{42} - 1}{.08}\right]$

$PMT = \$220{,}000 \div (304.244)$

$PMT = \$723.10$

b. $FVA_{42} = PMT \times \left[\dfrac{(1+.08)^{42} - 1}{.08}\right]$

$FVA_{42} = \$600 \times (304.244)$

$FVA_{42} = \$182{,}546.40$

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Present Value of Cash Flow Streams

P3-16. Given the mixed streams of cash flows shown in the following table, answer parts (a) and (b):

Year	Cash Flow Stream A	Cash Flow Stream B
1	$ 50,000	$ 10,000
2	40,000	20,000
3	30,000	30,000
4	20,000	40,000
5	10,000	50,000
Totals	$150,000	$150,000

a. Find the present value of each stream, using a 15 % discount rate.
b. Compare the calculated present values, and discuss them in light of the fact that the undiscounted total cash flows amount to $150,000 in each case.

A3-16.

a.

Cash Flow Stream	Year (t)	$CF_t \times (1+.15)^{-t}$ =	Present Value
A	1	$50,000 × .869565 =	$ 43,479
	2	$40,000 × .756144 =	30,246
	3	$30,000 × .657516 =	19,725
	4	$20,000 × .571753 =	11,435
	5	$10,000 × .497177 =	4,972
		Total =	$109,857

Cash Flow Stream	Year (t)	$CF_t \times (1+.15)^{-t}$ =	Present Value
B	1	$10,000 × .869565 =	$ 8,696
	2	$20,000 × .756144 =	15,123
	3	$30,000 × .657516 =	19,725
	4	$40,000 × .571753 =	22,870
	5	$50,000 × .497177 =	24,859
		Total =	$ 91,273

b. Cash flow stream A has a higher present value ($109,857) than cash flow stream B ($91,273) because cash flow stream A has larger cash flows in the early years when their present value is greater, while the smaller cash flows are received further in the future. Although both cash flow streams total $150,000 on an undiscounted basis, the large early-year cash flows of stream A result in its higher present value.

P3-17. As part of your personal budgeting process, you have determined that at the end of each of the next five years you will incur significant maintenance expenses on your home. You'd like to

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cover these expenses by depositing a lump sum in an account today that earns 8%. You will gradually draw down this account each year as maintenance bills come due.

End of Year	Budget Shortfall
1	$ 5,000
2	4,000
3	6,000
4	10,000
5	3,000

a. How much money must you deposit today to cover all of the expenses?
b. What effect does an increase in the interest rate have on the amount calculated in part (a)? Explain.

A3-17. a.

End of Year (t)	**Budget Shortfall × $(1 + .08)^{-t}$ =**	**Present Value**
1	$5,000 × .925926 =	$ 4,630
2	$4,000 × .857339 =	3,429
3	$6,000 × .793832 =	4,763
4	$10,000 × .735030 =	7,350
5	$3,000 × .680583 =	2,042
		$ 22,214

An initial deposit of $22,214 would be needed to fund the shortfall for the pattern shown in the table.

b. An increase in the earnings rate would reduce the amount calculated in part a.

P3-18. Ruth Nail receives two offers for her seaside home. The first offer is for $1 million today. The second offer is for an owner-financed sale with annual payments as follows:

End of Year	Payment
0 (Today)	$200,000
1	200,000
2	200,000
3	200,000
4	200,000
5	300,000

Assuming that Ruth earns a rate of 8 % on her investments, which offer should she take?

A3-18. PV of owner-financed sale:

End of Year (t)	**Cash Flow × $(1+.08)^{-t}$ =**	**Present Value**
0	$200,000 × 1.000000 =	$ 200,000
1	200,000 × .925926 =	185,186
2	200,000 × .857339 =	171,468
3	200,000 × .793832 =	158,766
4	200,000 × .735030 =	147,006
5	300,000 × .680583 =	204,175
		$1,066,601

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Ruth should take the second offer for the series of payments because it has a higher present value than the $1,000,000 payment today.

P3-19. Melissa Gould wants to invest today in order to assure adequate funds for her son's university education. She estimates that her son will need $20,000 in 18 years; $25,000 in 19 years; $30,000 in 20 years; and $40,000 in 21 years. How much does Melissa have to invest in a fund today if the fund earns the following interest rate?

a. 6 % per year with annual compounding
b. 6 % per year with quarterly compounding
c. 6 % per year with monthly compounding

A3-19. a. Amount required today with annual compounding (rate = .06; # periods = 1 × *n*)

$\$20,000 \times (1.06)^{-18}$	=	$ 7,007
$25,000 \times (1.06)^{-19}$	=	8,263
$30,000 \times (1.06)^{-20}$	=	9,354
$40,000 \times (1.06)^{-21}$	=	11,766
Total =		$36,390

b. Amount required today with quarterly compounding (rate = .06/4 = .015; # periods = 4 × *n*)

$\$20,000 \times (1.015)^{-72}$	=	$ 6,847
$25,000 \times (1.015)^{-76}$	=	8,063
$30,000 \times (1.015)^{-80}$	=	9,117
$40,000 \times (1.015)^{-84}$	=	11,453
Total =		$35,480

c. Amount required today with monthly compounding (rate= .06/12 = .005; # periods = 12 × *n*):

$\$20,000 \times (1.005)^{-216}$	=	$ 6,810
$25,000 \times (1.005)^{-228}$	=	8,018
$30,000 \times (1.005)^{-240}$	=	9,063
$40,000 \times (1.005)^{-252}$	=	11,382
Total =		$35,273

P3-20. Assume that you just won the state lottery. Your prize can be taken either in the form of $40,000 at the end of each of the next 25 years (i.e., $1 million over 25 years) or as a lump sum of $500,000 paid immediately.

a. If you expect to be able to earn 5% annually on your investments over the next 25 years, which alternative should you take? Why?
b. Would your decision in part (a) be altered if you could earn 7% rather than 5 % on your investments over the next 25 years? Why?
c. At approximately what interest rate would you be indifferent when choosing between the two plans?

A3-20. $PVA_n = \frac{PMT}{r} \times \left[1 - \frac{1}{(1+r)^n}\right]$

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a. PVA_{25} = (\$40,000 / 0.05) $\times$ [1 – (1 + .05)$^{-25}$]
PVA_{25} = \$800,000 $\times$ .704697
PVA_{25} = \$563,758
At 5%, taking the award as an annuity is better because its present value of \$563,578 is larger than the \$500,000 lump-sum amount.

b. PVA_{25} = (\$40,000 / 0.07) × [1 – (1 + .07)$^{-25}$]
PVA_{25} = \$571,429 $\times$ .815751
PVA_{25} = \$466,144

At 7%, taking the award as a lump sum is better because the present value of the annuity of \$466,144 is less than the \$500,000 lump-sum payment.

c. Because the annuity is worth more than the lump sum at 5% and less at 7%, try 6%:

PV_{25} = (\$40,000/0.06) $\times$ [1 – (1 + .06)$^{-25}$]
PV_{25} = \$666,667 $\times$ .767001
PV_{25} = \$511,335

The rate at which you would be indifferent is greater than 6%; about 6.25% using interpolation. Calculator solution is 6.24%.

P3-21. For the following questions, assume an end-of-year cash flow of \$250 and a 10% discount rate.
a. What is the present value of a single cash flow?
b. What is the present value of a 5-year annuity?
c. What is the present value of a 10-year annuity?
d. What is the present value of a 100-year annuity?
e. What is the present value of a \$250 perpetuity?
f. Do you detect a relationship between the number of periods of an annuity and its resemblance to a perpetuity? Explain.

A3-21. a. $PV = \$250 \times (1.10)^{-1} = \$227.27$

b. $PV = \$250 \times \left[\frac{1-(1.10)^{-5}}{0.10}\right] = \$947.70$

c. $PV = \$250 \times \left[\frac{1-(1.10)^{-10}}{0.10}\right] = \$1,536.14$

d. $PV = \$250 \times \left[\frac{1-(1.10)^{-100}}{0.10}\right] = \$2,499.82$

e. $PV = \frac{\$250}{0.10} = \$2,500.00$

f. As the number of periods in an annuity increases (as we move from part b to part d) and approaches infinity, the value of the annuity approaches the value of a perpetuity (part e).

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P3-22. Use the following table of cash flows to answer parts (a) and (b). Assume an 8% discount rate.

End of Year	Cash Flow
1	10,000
2	10,000
3	10,000
4	12,000
5	12,000
6	12,000
7	12,000
8	15,000
9	15,000
10	15,000

a. Solve for the present value of the cash flow stream by summing the present value of each individual cash flow.

b. Solve for the present value by summing the present value of the three separate annuities (one current and two deferred).

A3-22. a.

Year	Cash Flow	Present Value
1	$10,000	$ 9,259
2	10,000	8,573
3	10,000	7,938
4	12,000	8,820
5	12,000	8,167
6	12,000	7,562
7	12,000	7,002
8	15,000	8,104
9	15,000	7,504
10	15,000	6,948
Total		$79,877

b. $PV = \$10{,}000 \times \left[\frac{1-(1.08)^{-3}}{0.08}\right] + \$12{,}000 \times \left[\frac{1-(1.08)^{-4}}{0.08}\right] \times (1.08)^{-3} + \$15{,}000 \times \left[\frac{1-(1.08)^{-3}}{0.08}\right] \times (1.08)^{-7}$

PV = $25,771 + $31,551 + $22,556

PV = $79,878

P3-23. Consumer Insurance, Inc. sells extended warranties on appliances that provide coverage after the manufacturers' warranties expire. An analyst for the company forecasts that the company will have to pay warranty claims of $5 million per year for three years, with the first costs expected to occur four years from today. The company wants to set aside a lump sum today to cover these costs, and money invested today will earn 10%. How much does the company need to invest now?

A3-23. PV of deferred annuity* $= \$5{,}000{,}000 \times \left[\frac{1-(1.10)^{-3}}{.10}\right] \times (1.10)^{-3} = \$9{,}342{,}044$

* Note the present value of the three deposits is measured at the beginning of year 4, i.e., the end of year 3.

P3-24. Ed Lowman, the 20-year-old star opening batsman of his university cricket team, is approached about skipping his last two years of his four-year university degree and entering the professional cricket sports industry. Ed expects that his cricket career will be over by the time he is 32 years

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old. Talent scouts for regional cricket teams estimate that Ed could receive a signing bonus of $1 million today, along with a five-year contract for $3 million per year (payable at the end of each year). They further estimate that he could negotiate a contract for $5 million per year for the remaining seven years of his career. The scouts believe, however, that Ed will be a better selection for the Australian Test team if he improves by playing two more years of university cricket. If he stays at university, he is expected to receive a $2 million signing bonus in two years, along with a five-year contract for $5 million per year. After that, the scouts expect Ed to obtain a five-year contract for $6 million per year to take him into retirement. Assume that Ed can earn a 10% return over this time. Should Ed stay or go?

A3-24. PV of Ed entering the draft:

Signing bonus = $ 1,000,000

Initial contract $= \$3{,}000{,}000 \times \left[\dfrac{1-(1.10)^{-5}}{.10}\right] = \$\ 11{,}372{,}360$

Subsequent contract $= \$5{,}000{,}000\ x \left[\dfrac{1-(1.10)^{-7}}{.10}\right] \times (1.10)^{-5} = \$\ \underline{15{,}114{,}525}$

PV = $\underline{\underline{\$27{,}486{,}885}}$

PV of Ed playing out his eligibility:

Signing bonus $= \$2{,}000{,}000 \times (1.10)^{-2} = \$\ 1{,}652{,}893$

Initial contract $= \$5{,}000{,}000 \times \left[\dfrac{1-(1.10)^{-5}}{.10}\right] \times (1.10)^{-2} = \$\ 15{,}664{,}408$

Subsequent contract $= \$6{,}000{,}000 \times \left[\dfrac{1-(1.10)^{-5}}{.10}\right] \times (1.10)^{-7} = \underline{\$\ 11{,}671{,}638}$

PV = $\underline{\underline{\$28{,}988{,}939}}$

Because the PV of playing out his eligibility and then entering the draft is higher, Ed should stay in university.

P3-25. Matt Sedgwick, facilities and operations manager for the Darwin Dingoes professional rugby league team and stadium, has come up with an idea for generating income. Matt wants to expand the stadium by building skyboxes sold with lifetime (perpetual) season tickets. Each skybox will be guaranteed 10 season tickets at a cost of $200 per ticket per year for life. If each skybox costs $100,000 to build, what is the minimum selling price that Matt will have to charge for the skyboxes to break even, if the required return is 10 %?

A3-25. PV of ticket sales $= \dfrac{(\$200 \times 10)}{0.10} = \$20{,}000$

Breakeven selling price = $100,000 – $20,000 = $80,000.

P3-26. Jill Chu wants to choose the best of four immediate retirement annuities available to her. In each case, in exchange for paying a single premium today, she will receive equal annual end-of-year cash benefits for a specified number of years. She considers the annuities to be equally risky and is not concerned about their differing lives. Her decision will be based solely on the rate of return she will earn on each annuity. The key terms of each of the four annuities are shown in the following table.

Annuity	**Premium Paid Today**	**Annual Benefit**	**Life (years)**
A	$30,000	$3,100	20
B	25,000	3,900	10
C	40,000	4,200	15
D	35,000	4,000	12

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a. Calculate to the nearest 1 % the rate of return on each of the four annuities Jill is considering.
b. Given Jill's stated decision criterion, which annuity would you recommend?

A3-26. a. Loan A

$\$30{,}000 = \$3{,}100 \times (PVFA_{,r,\ 20\ yrs.})$
$9.677 = PVFA_{\ r\%,\ 20\ yrs.})$
$r = 8\%$
Calculator solution: 8.19%

Loan B

$\$25{,}000 = \$3{,}900 \times (PVFA_{r\%,10\ yrs.})$
$6.410 = PVIFA_{\ r\%,\ 10\ yrs.}$
$r = 9\%$
Calculator solution: 9.03%

Loan C

$\$40{,}000 = \$4{,}200 \times (PVFA_{r\%,\ 15\ yrs.})$
$9.524 = PVFA_{r\%,\ 15\ yrs.}$
$r = 6\%$
Calculator solution: 6.30%

Loan D

$\$35{,}000 = \$4{,}000 \times (PVFA_{r\%,\ 12\ yrs.})$
$8.75 = PVIFA_{r\%,\ 12\ yrs.}$
$r = 5\%$
Calculator solution: 5.23%

b. Annuity B gives the highest rate of return at 9% and would be the one selected based upon Jill's criteria.

P3-27. Evaluate each of the following three investments, each costing $1,000 today and providing the returns noted below, over the next five years.

Investment 1: $2,000 lump sum to be received in five years
Investment 2: $300 at the end of each of the next five years
Investment 3: $250 at the beginning of each of the next five years

a. Which investment offers the highest return?
b. Which offers the highest return if the payouts are doubled (i.e., $4,000, $600, and $500)?
c. What causes the big change in the returns on the annuities?

A3-27. a. Return on Investment # 1:

$\$2{,}000 = \$1{,}000 \times (1 + r)^5$
$2.0 = (1 + r)^5$
$(2.0)^{1/5} = 1 + r$
$1.1487 = 1 + r$
$r = 14.87\%$

Return on Investment # 2:

$$\$1{,}000 = \$300 \times \frac{1}{r} \times \left[1 - \frac{1}{(1+r)^n}\right]$$

Via trial and error or a financial calculator: $r \cong 15.24\%$

Return on Investment # 3:

$$\$1{,}000 = \$250 \times \frac{1}{r} \times \left[1 - \frac{1}{(1+r)^n}\right] \times (1 + r)$$

Via trial and error or a financial calculator: $r \cong 12.59\%$

Thus, Investment # 2 has the highest return.

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b. Return on Investment # 1:
$\$4{,}000 = \$1{,}000 \times (1 + r)^5$
$r = 31.95\%$

Return on Investment #2:
$$\$1{,}000 = 600 \times \frac{1}{r} \times \left[1 - \frac{1}{(1+r)^n}\right]$$
$r \cong 52.80\%$

Return on Investment # 3:

$$\$1{,}000 = \$500 \times \frac{1}{r} \times \left[1 - \frac{1}{(1+r)^n}\right] \times (1 + r)$$
$r \cong 92.76\%$

Thus, Investment # 3 has the highest return.

c. The annuities have much greater sensitivity because their intermediate cash flows are reinvested whereas the lump sum investment does not generate any intermediate cash flows that can be reinvested.

P3-28. Consider the following three investments of equal risk. Which offers the greatest rate of return?

	Investment		
End of Year	**A**	**B**	**C**
0	–$10,000	–$20,000	–$25,000
1	0	9,500	20,000
2	0	9,500	30,000
3	24,600	9,500	–12,600

A3-28. Note that all returns were calculated using a financial calculator.

Return on Investment A:

$\$24{,}600 = \$10{,}000 \times (1 + r)^3$
$r = 35\%$

Return on Investment B:

$$\$20{,}000 = \$9{,}500 \times \left[\frac{1 - (1 + r)^{-3}}{r}\right]$$
$r = 20\%$

Return on Investment C:

$\$25{,}000 = \$20{,}000 \times (1 + r)^{-1} + \$30{,}000 \times (1 + r)^{-2} - \$12{,}600 \times (1 + r)^{-3}$
$r = 40\%$

Advanced Applications of Time Value

P3-29. You plan to invest $2,000 in an individual retirement arrangement (IRA) today at a *stated interest rate* of 8%, which is expected to apply to all future years.

a. How much will you have in the account at the end of 10 years if interest is compounded as follows?

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(1) annually
(2) semiannually
(3) daily (assume a 365-day year)
(4) continuously

b. What is the *effective annual rate* (*EAR*) for each compounding period in part a?
c. How much greater will your IRA account balance be at the end of 10 years if interest is compounded continuously rather than annually?
d. How does the compounding frequency affect the future value and effective annual rate for a given deposit? Explain in terms of your findings in parts a–c.

A3-29. a. (1) $FV_{10} = \$2{,}000 \times (1.08)^{10}$
$FV_{10} = \$2{,}000 \times (2.159)$
$FV_{10} = \$4{,}318$

(2) $FV_{10} = \$2{,}000 \times (1.04)^{20}$
$FV_{10} = \$2{,}000 \times (2.191)$
$FV_{10} = \$4{,}382$

(3) $FV_{10} = \$2{,}000 \times (1.00022)^{3600}$
$FV_{10} = \$2{,}000 \times (2.208)$
$FV_{10} = \$4{,}416$

(4) $FV_{10} = \$2{,}000 \times e^{.8}$
$FV_{10} = \$2{,}000 \times (2.226)$
$FV_{10} = \$4{,}452$

b. (1) $EAR = (1 + .08/1)^1 - 1$
$EAR = (1 + .08)^1 - 1$
$EAR = (1.08) - 1$
$EAR = .08 = 8\%$

(2) $EAR = (1 + .08/2)^2 - 1$
$EAR = (1 + .04)^2 - 1$
$EAR = (1.0816) - 1$
$EAR = .0816 = 8.16\%$

(3) $EAR = (1 + .08/360)^{360} - 1$
$EAR = (1 + .00022)^{360} - 1$
$EAR = (1.0824) - 1$
$EAR = .0824 = 8.24\%$

(4) $EAR = (e^k - 1)$
$EAR = (e^{08} - 1)$
$EAR = (1.0833 - 1)$
$EAR = .0833 = 8.33\%$

c. The IRA account balance at the end of 10 years will be \$134 (\$4,452 – \$4,318) larger with continuous rather than annual compounding.
d. The more frequently interest is compounded at a given nominal annual rate, the greater the future value and the higher the effective annual rate, EAR.

P3-30. Binh Tran has shopped around for the best interest rates for his investment of \$10,000 over the next year. He has found the following:

Stated Rate	**Compounding**
6.10%	annual
5.90%	semiannual
5.85%	monthly

a. Which investment offers Binh the highest effective annual rate of return?
b. Assume that Binh wants to invest his money for only six months and the annual compounded rate of 6.10 per cent is not available. Which of the remaining investments should Binh choose?

A3-30. a.

Nominal Rate	**Compounding**	**Effective Annual Rate**
6.10%	Annual	6.10%
5.90%	Semiannual	$\left(1+\frac{0.59}{2}\right)^2 - 1 = 5.99\%$

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5.85%	Monthly	$\left(1+\frac{.0585}{12}^{12}\right)-1=6.01\%$

The annual-compounded rate of 6.10% is also the highest effective rate

b. He would prefer the monthly compounding case because it offers a slightly higher effective annual rate of interest.

P3-31. Answer parts a–c for each of the following cases.

Case	Amount of Initial Deposit ($)	Stated Annual Rate, *r* (%)	Compounding Frequency, *m* (times/year)	Deposit Period (years)
A	2,500	6	2	5
B	50,000	12	6	3
C	1,000	5	1	10
D	20,000	16	4	6

a. Calculate the future value at the end of the specified deposit period.
b. Determine the effective annual rate (*EAR*).
c. Compare the stated annual rate (*r*) to the effective annual rate (*EAR*). What relationship exists between compounding frequency and the stated and effective annual rates?

A3-31. a. Compounding Frequency: $FV_n = PV \times (1 + r)^n$

A. $FV_5 = \$2{,}500 \times (1.03)^{10}$
$FV_5 = \$2{,}500 \times (1.344)$
$FV_5 = \$3{,}360$

B. $FV_3 = \$50{,}000 \times (1.02)^{18}$
$FV_3 = \$50{,}000 \times (1.428)$
$FV_3 = \$71{,}412$

C. $FV_{10} = \$1{,}000 \times (1.05)^{10}$
$FV_{10} = \$1{,}000 \times (1.629)$
$FV_{10} = \$1{,}629$

D. $FV_6 = \$20{,}000 \times (1.04)^{24}$
$FV_6 = \$20{,}000 \times (2.563)$
$FV_6 = \$51{,}266$

b. Effective Annual Rate: EAR = $\left(1+\frac{r}{m}\right)^m - 1$

A. EAR = $(1 + .06/2)^2 - 1$
EAR = $(1 + .03)^2 - 1$
EAR = $(1.061) - 1$
EAR = .061 = 6.1%

B. EAR = $(1 + .12/6)^6$
EAR = $(1 + .02)^6 - 1$
EAR = $(1.126) - 1$
EAR = .126 = 12.6%

C. EAR = $(1 + .05/1)^1 - 1$
EAR = $(1 + .05)^1 - 1$
EAR = $(1.05) - 1$
EAR = .05 = 5%

D. EAR = $(1 + .16/4)^4 - 1$
EAR = $(1 + .04)^4 - 1$
EAR = $(1.170) - 1$
EAR = .17 = 17%

c. The effective rates of interest rise with increasing compounding frequency.

P3-32. Tara Cutler is newly married and is now preparing a surprise gift of a trip to Europe for her husband on their tenth anniversary. Tara plans to invest $5,000 per year until that anniversary and plans to make her first $5,000 investment on their first anniversary. If she earns an 8% rate on her investments, how much will she have saved for their trip if the interest is compounded in each of the following ways?

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a. annually
b. quarterly
c. monthly

A3-32. a. Effective rate = nominal rate = 8%

$$FVA_{10} = \$5,000 \times \left[\frac{(1.08)^{10} - 1}{0.08}\right] = \$72,433$$

b. Effective rate = $(1 + \frac{.08}{4})^4 - 1 = 8.24\%$

$$FVA_{10} = \$5,000 \times \left[\frac{(1.0824)^{10} - 1}{0.0824}\right] = \$73,264$$

c. Effective rate = $(1 + \frac{.08}{12})^{12} - 1 = 8.30\ \%$

$$FVA = \$5,000 \times \frac{(1.083)^{10} - 1}{0.083} = \$73,473$$

P3-33. John Tye was hired as the new company finance analyst at I-Ell Enterprises and received his first assignment. John is to take the $25 million in cash received from a recent divestiture, to use part of these proceeds to retire an outstanding $10 million bond issue and to use the remainder to repurchase ordinary shares. However, the bond issue cannot be retired for another two years. If John can place the funds necessary to retire this $10 million debt into an account earning a 6% annual return compounded monthly, how much of the $25 million remains to repurchase shares?

A3-33. PV of debt obligation = $\$10,000,000 \times (1.005)^{-24} = \$8,871,857$

Funds remaining for share repurchase = $25,000,000 – $8,871,857 = $16,128,143.

P3-34. Find the present value of a 3-year, $20,000 ordinary annuity deposited into an account that pays 12% annual interest, compounded *monthly*. Solve for the present value of the annuity in the following ways:
a. as three single cash flows discounted at the *stated annual rate* of interest
b. as three single cash flows discounted at the appropriate *effective annual rate* of interest
c. as a 3-year annuity discounted at the *effective annual rate* of interest

A3-34. a. $PV = \$20,000 \times (1.01)^{-12} + \$20,000 \times (1.01)^{-24} + \$20,000 \times (1.01)^{-36} = \$47,479$

b. Effective rate = $(1 + \frac{.12}{12})^{12} - 1 = 12.68\%$

$PV = \$20,000 \times (1.1268)^{-1} + \$20,000 \times (1.1268)^{-2} + \$20,000 \times (1.1268)^{-3} = \$47,481$ (rounding)

c. $PVA_3 = \$20,000 \times \left[\frac{1 - (1.1268)^{-3}}{0.1268}\right] = \$47,481$ (rounding)

P3-35. Determine the annual payment required to fund a future annual annuity of $12,000 per year. You will fund this future liability over the next five years, with the first payment to occur one year

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from today. The future $12,000 liability will last for four years, with the first payment to occur seven years from today. If you can earn 8% on this account, how much will you have to deposit each year over the next five years to fund the future liability?

A3-35. Present Value of Future Liability = Present Value of Funding Annuity

$$\left[\frac{1-(1.08)^{-4}}{.08}\right] \times (1.08)^{-6} \times \$12{,}000 = \left[\frac{1-(1.08)^{-5}}{.08}\right] \times \text{Funding annuity}$$

$25,046.42 = 3.99271 × Funding annuity
Funding annuity = $6,273.04

P3-36. Mary Chong, capital expenditure manager for PDA Manufacturing, knows that her company is facing a series of monthly expenses associated with installation and calibration of new production equipment. The company has $1 million in a bank account right now that it can draw on to meet these expenses. Funds in this account earn 6% interest annually, with monthly compounding. Ms Chong is preparing a budget that will require the company to make equal monthly deposits into their bank account, starting next month, to ensure that they can pay the repair costs they anticipate over the next 24 months (shown as follows). How much should the monthly bank deposit be?

Months	Repair Costs per Month
1–4	$100,000
5–12	$200,000
13–24	$500,000

A3-36. Present Value of Future Liabilities = Present Value of Required Funding Annuity
(rate = .06/12 = .005; # periods = 12 × *n*)

$$\$500{,}000 \times \left[\frac{1-(1.005)^{-4}}{.005}\right]$$

$$+ \$250{,}000 \times \left[\frac{1-(1.005)^{-8}}{.005}\right] \times (1.005)^{-4} = \text{Annuity} \times \left[\frac{1-(1.005)^{-24}}{.005}\right] + \$1{,}000{,}000$$

$$+\$100{,}000 \times \left[\frac{1-(1.005)^{-12}}{.005}\right] \times (1.005)^{-12}$$

$4,986,751 = Annuity × 22.5629 + 1,000,000

$3,986,750 = Annuity × 22.5629

Annuity = $176,695

P3-37. Craig and LaDonna Allen are trying to establish a university fund for their son Spencer, who just turned three today. They plan for Spencer to withdraw $10,000 on his eighteenth birthday and $11,000, $12,000, and $15,000 on his subsequent birthdays to cover the expected four years of his university education. They plan to fund these withdrawals with a 10-year annuity, with the first payment to occur one year from today, and expect to earn an average annual return of 8 %.

a. How much will the Allens have to contribute each year to achieve their goal?
b. Create a schedule showing the cash inflows (including interest) and outflows of this fund. How much remains on Spencer's twenty-first birthday?

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A3-37. a. Amount needed at Spencer's 13th birthday (in 10 years):

$= \$10{,}000 \times (1.08)^{-5} + \$11{,}000 \times (1.08)^{-6} + \$12{,}000 \times (1.08)^{-7} + \$15{,}000 \times (1.08)^{-8}$

$= \$6{,}805.83 + \$6{,}931.87 + \$7{,}001.88 + \$8{,}104.03 = \$28{,}843.61$

$$\text{Funding payment} \times \left[\frac{(1+.08)^{10}-1}{.08}\right] = \$28{,}843.61$$

Funding payment × 14.486562 = $28,843.61

$$\text{Funding payment} = \frac{\$28{,}843.61}{14.486562} = \$1{,}991.06$$

b.

End of Spencer's Birthday Year	Deposit (Withdrawal)	Beginning Balance	Ending Balance (Begin Balance × 1.08)
4	$ 1,991.06	$ 1,991.06	$ 2,150.34
5	1,991.06	4,141.40	4,472.72
6	1,991.06	6,463.78	6,980.88
7	1,991.06	8,971.94	9,689.69
8	1,991.06	11,680.75	12,615.21
9	1,991.06	14,606.27	15,774.78
10	1,991.06	17,765.84	19,187.10
11	1,991.06	21,178.16	22,872.42
12	1,991.06	24,863.48	26,852.56
13	1,991.06	28,843.62	31,151.11
14	0	31,151.11	33,643.20
15	0	33,643.20	36,334.65
16	0	36,334.65	39,241.43
17	0	39,241.43	42,380.74
18	(10,000)	32,380.74	34,971.20
19	(11,000)	23,971.20	25,888.90
20	(12,000)	13,888.90	15,000.00
21	(15,000)	0	

Nothing remains in the account after the $15,000 withdrawal is made on Spencer's 21st birthday.

P3-38. Joan Messineo borrowed $15,000 at a 14 % annual interest rate to be repaid over three years. The loan is amortised into three equal annual end-of-year payments.

a. Calculate the annual end-of-year loan payment.
b. Prepare a loan amortisation schedule showing the interest and principal breakdown of each of the three loan payments.
c. Explain why the interest portion of each payment declines with the passage of time.

A3-38. a.

$$\text{PMT} = \$15{,}000 \div \left[\frac{1-(1.14)^{-3}}{0.14}\right]$$

PMT = $15,000 ÷ 2.321632

PMT = $6,460.97

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b.

End of Year	Loan Payment	Beginning of Year Principal	Payment Interest	Payment Principal	End of Year Principal
1	$ 6,460.97	$15,000.00	$2,100.00	$4,360.97	$10,639.03
2	6,460.97	10,639.03	1,489.46	4,971.51	5,667.52
3	6,460.97	5,667.52	793.45	5,667.52	0.00

c. Through annual end-of-year payments, the principal balance of the loan is declining, causing less interest to be accrued on the balance in each subsequent year.

P3-39. You are planning to purchase a caravan for $40,000, and you have $10,000 to apply as a down payment. You may borrow the remainder under the following terms: a 10-year loan with semiannual repayments and a stated interest rate of 6 %. You intend to make $6,000 payments, applying the excess over your required payment to the reduction of the principal balance.

a. Given these terms, how long (in years) will it take you to fully repay your loan?
b. What will be your total interest cost?
c. What would your interest cost be if you made no prepayments and repaid your loan by strictly adhering to the terms of the loan?

A3-39. a. Required Payment:

$$PMT = \frac{\$30{,}000}{\frac{[1-(1+\{.06/2\}^{-2\times 10}]}{.06/2}} = \frac{\$30{,}000}{\frac{1-(1.03)^{-20}}{.03}} = \$2{,}016.47$$

Amortisation Schedule

Period	Beginning Balance	Payment	Interest (.03 × Principal)	Principal	Prepay	Ending Balance
1	$30,000.00	$2,016.47	$ 900.00	$1,116.47	$3,983.53	$24,900.00
2	24,900.00	2,016.47	747.00	1,269.47	3,983.53	19,647.00
3	19,647.00	2,016.47	589.41	1,427.06	3,983.53	14,236.41
4	14,236.41	2,016.47	427.09	1,589.38	3,983.53	8,663.50
5	8,663.50	2,016.47	259.91	1,756.56	3,983.53	2,923.41
6	2,923.41	2,016.47	87.70	1,928.77	994.64	0
			$3,011.11			

The loan will be paid off in 6 periods or 3 years.

b. Total interest cost = $3,011.11

c. Total interest cost with no prepayments:
$20 \times \$2{,}016.47 - \$30{,}000 = \$10{,}329.40$

P3-40. Use a spreadsheet to create amortisation schedules for the following five scenarios. What happens to the total interest paid under each scenario?

a. **Scenario 1**:
Loan amount: $1 million
Annual rate: 5 per cent
Term: 360 months
Prepayment: $0

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b. **Scenario 2**: Same as 1, except annual rate is 7 %
c. **Scenario 3**: Same as 1, except term is 180 months
d. **Scenario 4**: Same as 1, except prepayment is $250 per month
e. **Scenario 5**: Same as 1, except loan amount is $125,000

A3-40. a.

Period	Beginning Balance	Payment	Interest	Principal	Ending Balance
1	$1,000,000.00	$5,368.22	$4,166.67	$1,201.55	$998,798.45
2	$998,798.45	$5,368.22	$4,161.66	$1,206.56	$997,591.89
3	$997,591.89	$5,368.22	$4,156.63	$1,211.58	$996,380.31

**** Years in between are not shown for practical purposes ****

Period	Beginning Balance	Payment	Interest	Principal	Ending Balance
358	$15,971.37	$5,368.22	$66.55	$5,301.67	$10,669.70
359	$10,669.70	$5,368.22	$44.46	$5,323.76	$5,345.94
360	$5,345.94	$5,368.22	$22.27	$5,345.94	$0.00
Total			$932,557.84	$1,000,000.00	

b.

Period	Beginning Balance	Payment	Interest	Principal	Ending Balance
1	$1,000,000.00	$6,653.02	$5,833.33	$819.69	$999,180.31
2	$999,180.31	$6,653.02	$5,828.55	$824.47	$998,355.84
3	$998,355.84	$6,653.02	$5,823.74	$829.28	$997,526.55

**** Years in between are not shown for practical purposes ****

Period	Beginning Balance	Payment	Interest	Principal	Ending Balance
358	$19,728.46	$6,653.02	$115.08	$6,537.94	$13,190.52
359	$13,190.52	$6,653.02	$76.94	$6,576.08	$6,614.44
360	$6,614.44	$6,653.02	$38.58	$6,614.44	$0.00
Total			$1,395,088.98	$1,000,000.00	

c.

Period	Beginning Balance	Payment	Interest	Principal	Ending Balance
1	$1,000,000.00	$7,907.94	$4,166.67	$3,741.27	$996,258.73
2	$996,258.73	$7,907.94	$4,151.08	$3,756.86	$992,501.87
3	$992,501.87	$7,907.94	$4,135.42	$3,772.51	$988,729.36

**** Years in between are not shown for practical purposes ****

Period	Beginning Balance	Payment	Interest	Principal	Ending Balance
178	$23,527.47	$7,907.94	$98.03	$7,809.91	$15,717.57
179	$15,717.57	$7,907.94	$65.49	$7,842.45	$7,875.12
180	$7,875.12	$7,907.94	$32.81	$7,875.12	$0.00
Total			$423,428.53	$1,000,000.00	

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d.

Period	Beginning Balance	Payment	Interest	Principal	Prepay	Ending Balance
1	$1,000,000.00	$5,368.22	$4,166.67	$1,201.55	$250.00	$998,548.45
2	$998,548.45	$5,368.22	$4,160.62	$1,207.60	$250.00	$997,090.85
3	$997,090.85	$5,368.22	$4,154.55	$1,213.67	$250.00	$995,627.18

**** Years in between are not shown for practical purposes ****

*** With the prepayment of $250.00 per month you'll have paid the loan in full by month 326***

324	$13,877.39	$5,368.22	$57.82	$5,310.39	$250.00	$8,317.00
325	$8,317.00	$5,368.22	$34.65	$5,333.56	$250.00	$2,733.43
326	$2,733.43	$5,368.22	$11.39	$2,733.43	$0.00	$0.00
Total			$828,665.10	$918,750.00	$81,250.00	
				Principal + Prepay	$1,000.000.00	

e.

Period	Beginning Balance	Payment	Interest	Principal	Ending Balance
1	$125,000.00	$671.03	$520.83	$150.19	$124,849.81
2	$124,849.81	$671.03	$520.21	$150.82	$124,698.99
3	$124,698.99	$671.03	$519.58	$151.45	$124,547.54

**** Years in between are not shown for practical purposes ****

358	$1,996.42	$671.03	$8.32	$662.71	$1,333.71
359	$1,333.71	$671.03	$5.56	$665.47	$668.24
360	$668.24	$671.03	$2.78	$668.24	$0.00
Total			$116,569.73	$125,000.00	

P3-41. You are the pension fund manager for Tanju's Toffees, and your CFO has just made a request of you. The CFO wants to know the minimum annual return required on the pension fund in order to make all required payments over the next five years and not diminish the existing asset base. The fund currently has assets of $500 million.

a. Determine the required return if outflows are expected to exceed inflows by $50 million per year.

b. Determine the required return with the following fund cash flows.

End of Year	Inflows	Outflows
1	$55,000,000	$100,000,000
2	60,000,000	110,000,000
3	60,000,000	120,000,000
4	60,000,000	135,000,000
5	64,000,000	145,000,000

c. Consider the cash flows in part b. What will happen to your asset base if you earn 10%? 20 %?

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A3-41. a. $\frac{\$50,000,000}{\$500,000,000} = 10\%$

Net Outflows*

b. $\$500,000,000 = \$\ 45,000,000 \times (1+r)^{-1}$
$+\ 50,000,000 \times (1+r)^{-2}$
$+\ 60,000,000 \times (1+r)^{-3}$
$+\ 75,000,000 \times (1+r)^{-4}$
$+\ 81,000,000 \times (1+r)^{-5}$
$+\ 500,000,000 \times (1+r)^{-5}$

* Outflows – inflows in each year.

Using trial and error techniques or a financial calculator results in a required rate of return of 12% to fund the difference between inflows and outflows over the next five years without depleting the asset base of $500,000,000.

c. Earning less than 12% will diminish the asset base while earning greater than 12% will grow the asset base.

P3-42. You plan to start saving for your son's university education. He will begin university when he turns 18 years old and will need $4,000 then and in each of the following three years. You will make a deposit at the end of this year in an account that pays 6 % compounded annually, and an identical deposit at the end of each year, with the last deposit occurring when he turns 18. If an annual deposit of $1,484 will allow you to reach your goal, how old is your son now?

A3-42. PV of funding annuity = PV of university expense annuity

$$\$1,484 \times \left[\frac{1-(1.06)^{-n}}{0.06}\right] = \$4,000 \times \left[\frac{1-(1.06)^{-4}}{0.06}\right] \times (1.06)^{-[n-1]}$$

$$\$1,484 \times \left[\frac{1-(1.06)^{-n}}{0.06}\right] = \$13,860.42 \times (1.06)^{-[n-1]}$$

Via trial and error or a financial calculator, solve for n = 8.
Thus, your son will turn 18 eight years from today and is 10 years old now.

Answer to MiniCase

Present Value

Casino.com Corporation is building a $25 million office building in Adelaide and is financing the construction at an 80 % loan-to-value ratio, where the loan is in the amount of $20,000,000. This loan has a ten-year maturity, calls for monthly payments, and is contracted at an interest rate of 8%.

Assignment

Using the above information, answer the following questions.

1. What is the monthly payment?
2. How much of the first payment is interest?
3. How much of the first payment is principal?
4. How much will Casino.com Corporation owe on this loan after making monthly payments for three years (the amount owed immediately after the thirty-sixth payment)?
5. Should this loan be refinanced after three years with a new seven-year 7 per cent loan, if the cost to refinance is $250,000? To make this decision, calculate the new loan payments and then the present value of the difference in the loan payments.

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6. Returning to the original ten-year 8 per cent loan, how much is the loan payment if these payments are scheduled for quarterly rather than monthly payments?
7. For this loan with quarterly payments, how much will Casino.com Corporation owe on this loan after making quarterly payments for three years (the amount owed immediately after the twelfth payment)?
8. What is the annual percentage rate on the original ten-year 8 % loan?
9. What is the *effective annual rate (EAR)* on the original ten-year 8 % loan?

Answers

1. PV = 20,000,000
   n = 10 × 12 = 120
   i = 8/12 = 0.6667

   Monthly payment = PMT = $242,655.19

2. Interest = Loan amount × Monthly interest rate
   = $20,000,000 × 8%/12
   = $133,333.33

3. Principal first month = Monthly payment – Interest first month
   = $242,655.19 – $133,333.33
   = $109,321.86

4. 7 Years × 12 Months = 84 Payments remaining
   PMT = $242,655.19
   n = 84
   i = 8%/12 = 0.6667
   Principal after 3 years = PV = $15,568,577.62

5. New loan payments:
   PV = $15,568,577.62
   n = 7 Years × 12 Months = 84 Months
   i = 7%/12 = 0.5833
   PMT = $234,971.55

   Difference in loan payments = $242,655.19 – $234,971.55 = $7683.63

   Present value of difference:
   PMT = $7,683.63
   n = 7 Years × 12 Months = 84
   i = 7%/12 = 0.5833
   PV = $509,096.48

   Since savings from refinancing is greater than the cost to refinance ($509,096.48 > $250,000), then the loan should be refinanced.

6. PV = $20,000,000
   n = 10 Years × 4 Quarters = 40 Quarters
   i = 8%/4 = 2%
   Quarterly payments = PMT = $731,114.96

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7. $n = 7$ Years $\times$ 4 Quarters = 28
   $i = 8\%/4 = 2\%$
   PMT = \$731,114.96
   Amount owed after 3 years = PV = \$15,559,056.50

8. Annual percentage rate = 8%

9. EAR $= (1 + \underline{r})^{m} - 1$

$$\frac{1}{r} \times \left[1 - \frac{1}{(1+r)^{n}}\right] = \frac{1}{0.055} \times \left[1 - \frac{1}{(1.055)^{5}}\right] = 4.2703$$

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